

A Unified Approach to Supersymmetry Breaking

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Abstract

General formulae for the soft SUSY breaking terms, valid in any SUGRA context, were derived in the mid-nineties. Since SUSY is not expected to have quantum anomalies, they should be valid in the quantum theory and be RG invariant down to the soft SUSY breaking scale. This observation enables us to give a uniform treatment of all phenomenological models for SUSY breaking and transmission, such as AMSB, GMSB, etc. In particular we find that the much discussed RG invariant formulae for soft SUSY breaking parameters in AMSB, effectively depend on a strong assumption of factorizability of the matter Kaehler metric. We then argue that there is no necessity for having ad hoc constructions such as mAMSB to counteract the negative squared slepton mass problem, since the natural framework that emerges in a sequestered model is one in which gaugino masses are as in AMSB, and the other soft terms are generated by RG running as in gaugino mediation.

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1 Introduction

A theory of supersymmetry (SUSY) breaking necessarily involves supergravity (SUGRA). The reason is that spontaneous supersymmetry breaking in a global SUSY theory leads to a positive vacuum energy at the scale of supersymmetry breaking. Given the limits on superpartner masses this is necessarily many orders of magnitude greater than the observed value of the cosmological constant (CC). In SUGRA by contrast the CC can in principle be fine-tuned to values parametrically below the SUSY breaking scale.

However SUGRA is not an ultra-violet complete theory. It must necessarily be contained within a consistent theory of quantum gravity. It is widely expected that string theory is such a theory. In any case whether or not string theory is the UV completion of SUGRA, it is clear that some more fundamental theory must replace SUGRA at some scale $\Lambda \leq M_P$ the Planck scale, where even the notion of a smooth metric background is expected to breakdown. The theory below such a scale may be expanded in terms of the number of derivatives scaled by Λ .

General formulae for the so-called soft terms which characterize the supersymmetry breaking parameters in the MSSM have been discussed many years ago [1, 2]. As was stressed in the first of these references these formulae are renormalization group invariant at least down to the soft SUSY breaking scale. While they were originally discussed within the context of string phenomenology, they are valid regardless of the validity of string theory. The necessary criteria are the following:

- There is some UV scale (say $\Lambda < M_P$) up to which there is a low energy (4D) SUGRA description of nature.
- At the relevant energy scale E , $E/\Lambda \ll 1$, the SUGRA theory can be truncated to a two derivative theory. For consistency this also requires the restriction $F/\Lambda^2 \ll 1$. In string theory based models for example, $\Lambda \lesssim M_{KK}$ where M_{KK} is the lowest Kaluza-Klein mass.
- Under these conditions the effective theory - including quantum corrections can be described by a real analytic Kaehler potential K , an analytic superpotential W and an analytic gauge coupling function f .
- The set of chiral superfields in the theory can be broken up into two classes. 1) A set of (gauge neutral) fields which will have large ground state values at the minimum of the scalar potential, and may have non-zero F-terms, and 2) a set of fields which will have essentially zero vacuum expectation values (vev's) and F-terms. Here large means values that are at least a significant fraction of Λ . Actually in all but GMSB these values will be at least of order M_P . Fields in category 1) will be referred to as moduli (even though they need not have anything to do with the moduli of string theory).

In GMSB type models the SUSY breaking sector has nothing to do with the string theory moduli. It is usually taken to be some O'Raifeartaigh type model. Such a model may be put in a canonical form, with heavy fields which do not participate in SUSY breaking, and a single (light) field X which develops an F-term [3]. If the theory is embedded in a string theory it is first necessary to integrate out all the closed string moduli in a supersymmetric Minkowski background, so that one can have a SUGRA formulation at some energy scale lower than the mass of the lightest closed string modulus. (see for example [4]). GMSB has in addition to this SUSY breaking sector, a messenger sector (in direct mediation models it may be part of the breaking sector), that couples to both the SUSY breaking sector and the standard model gauge fields. The mass scale of the

messengers is (essentially) given by X_0 , the vev of the SUSY breaking field. Thus the consistency of the two derivative theory below the messenger scale then requires that $|\frac{F^X}{X_0}| \ll 1^1$.

A bottom up approach to soft mass formulae focussing mostly on the quantum effects associated with gauge mediated supersymmetry breaking (GMSB) was published [5] several years after [1]. Here the supersymmetry breaking was introduced as a non-dynamical (or spurion) field with a non-zero F-term. One of the main points of our paper, is to emphasize not only that these arguments are implicit in [1, 2], but also that the formulae given there, give a more powerful method of deriving all the quantum effects. In particular it shows how all of the currently popular models of SUSY breaking emerge in a natural fashion from different assumptions about the moduli dependence of the matter metric.

In the rest of the paper we will focus on two derivative terms under the assumption that the supersymmetry breaking is small in the above sense that $F/\Lambda^2 \ll 1$, where Λ is the mass of the lowest scale that has been integrated out. In the next section we will discuss the general framework of two derivative SUGRA theories and then how different popular models of SUSY breaking emerge from this general framework. In particular we will discuss the validity of AMSB arguments. Then we will discuss how GMSB fits into this framework. Finally we will discuss the natural replacement of AMSB within the context of sequestered theories. This is the mechanism which has been called gaugino anomaly mediation (inoAMSB), discussed in [6, 7]. We end with a brief summary of our results.

2 Supergravity formalism and the soft SUSY breaking terms

The most general manifestly supersymmetric action for chiral scalar fields Φ coupled to supergravity and gauge fields, when restricted to no more than two derivatives, can be expressed in terms of three functions. i) The (real analytic) Kaehler potential $K(\Phi, \bar{\Phi})$, the analytic superpotential $W(\Phi)$, and the (analytic) gauge coupling function (or functions if there is more than one simple group factor) $f(\Phi)$. (see for example [8],[9]).

The action of superspace supergravity coupled to chiral scalars and Yang-Mills fields is (following the notation and conventions of [8] but with $M_P = \kappa^{-1} = 1$),

$$S = -3 \int d^6 z 2\mathcal{E} \left(-\frac{\bar{\nabla}^2}{4} + 2R \right) \exp \left[-\frac{1}{3} K(\Phi, \bar{\Phi}; Q, \bar{Q} e^{2V}) \right] + \left(\int d^6 z 2\mathcal{E} [W(\Phi, Q) + \frac{1}{4} f(\Phi) \mathcal{W}^a \mathcal{W}^a] + h.c. \right). \quad (1)$$

The arguments given in [8],[9] (or for that matter the original arguments of [10]) imply that the effective action at some scale after including all effects (classical and quantum) coming from integrating out states at higher scales, must still be of this form. In particular it remains an action that is determined by three functions K, W, f which are just dependent on the physical chiral fields Φ . All that can change (relative to some 'classical' expression) is the functional form of these superfields.

The main point of this paper is that any low energy physical effect should be obtainable from this action as long as the restrictions of the supersymmetric derivative expansion discussed in the introduction are satisfied. This means that once the functional form of K, W and f are given (including the quantum corrections) one should be able to read off the physical masses

¹This also means that the component calculations of gaugino and scalar masses in GMSB should only be trusted at lowest non-trivial order in this parameter since the starting point has effectively ignored the higher order terms.

and couplings of the theory (at the scale at which we expect these forms to be valid), from the expression in component form for the above action that is given in (for instance) Appendix G of [8].

2.1 General Expressions for soft terms and RG invariance

What is of most interest for us in the context of (low energy) SUSY breaking is the boundary values of the soft masses and couplings, which in the context of the MSSM will become the parameters of phenomenological interest. The theory above has a set of gauge neutral fields $\Phi = \{\Phi^A\}$, which in a string theory context for instance, would be identified as the moduli determining the size and shape of the internal 6D manifold as well as the string coupling. In general we need to find the point at which these are stabilized in a SUSY breaking fashion, and is such that none of the charged fields $Q = \{Q^a\}$ get a vacuum value. If one finds such a minimum then the soft masses are obtained in the manner described below [1].

We expand the superpotential and the Kaehler potential in powers of the charged fields, i.e. we write

$$W = \hat{W}(\Phi) + \frac{1}{2}\tilde{\mu}_{ab}(\Phi)Q^aQ^b + \frac{1}{6}\tilde{Y}_{abc}(\Phi)Q^aQ^bQ^c + \dots, \quad (2)$$

$$K = \hat{K}(\Phi, \bar{\Phi}) + Z_{a\bar{b}}(\Phi, \bar{\Phi})Q^aQ^{\bar{b}} + [X_{ab}(\Phi, \bar{\Phi})Q^aQ^b + h.c.] + \dots \quad (3)$$

Then one may easily compute the soft masses from the well known expression for the scalar potential in supergravity and get [1][2],

$$(m^2)_a^{a'} \equiv Z^{a'\bar{b}}m_{a\bar{b}}^2 = \frac{1}{3}(2V_0 + F^AF^{\bar{B}}\hat{K}_{A\bar{B}})\delta_a^{a'} - F^AF^{\bar{B}}R_{A\bar{B}a\bar{b}}Z^{a'\bar{b}} \quad (4)$$

$$= \frac{1}{3}(2V_0 + F^AF^{\bar{B}}\hat{K}_{A\bar{B}})\delta_a^{a'} - F^AF^{\bar{B}}\partial_A(Z^{a'\bar{b}}\partial_{\bar{B}}Z_{a\bar{b}}) \quad (5)$$

Note that these expressions are written for the canonically normalized fields so that these expressions are valid for the normalized squared mass matrix. Note also that while the first term is proportional to the unit matrix, the second is not necessarily so, and hence is in general a potential source of flavor violation.

Now while (1) is manifestly (off-shell) supersymmetric, the component form (given for instance in Appendix G of [8]) only has on-shell supersymmetry. In fact in arriving at the latter a series of (super) Weyl transformations and field redefinitions of chiral multiplets has been performed. This is necessary in order to get to the Einstein frame for (super) gravity and Kaehler normalization for the chiral fields (with for instance the scalar field kinetic term being of the form $K_{a\bar{b}}\partial_\mu\phi^a\partial^\mu\bar{\phi}^{\bar{b}}$). Now in the quantum theory these transformations do not leave the measure invariant and there is an anomaly. However as usual this anomaly just changes the gauge coupling function (at the two derivative level), and has no effect on the Kaehler metric. Hence the above formula (4) remains valid in the quantum theory - assuming of course that the appropriate K, W, f are used. For instance the dilaton component of the Kaehler potential of the heterotic string is a term of the form $K \sim -\ln(S + \bar{S})$. Due to string loop effects this term gets changed to $K \sim -\ln(S + \bar{S} - \Delta(M, \bar{M})/16\pi^2)$ [11]. This will obviously change the curvature term in (4) - but this has nothing to do with an anomaly. Similar considerations apply to the expressions for the $\mu, B\mu$ and A terms given in [1][2].

The gauge coupling function on the other hand does experience an anomaly, since the above field redefinitions give contributions to the measure which are of the form $\exp\{\# \int \tau WW + h.c.\}$, where τ is some (chiral) superfield transformation parameter. In particular this means that the

gauge coupling function $g_{phys}(\Phi, \bar{\Phi})$ and the gaugino mass $M(\Phi, \bar{\Phi})$ (after supersymmetry breaking i.e. $F^A \neq 0$ for at least one value of A), are given by

$$\frac{1}{g_{phys}^2} = \Re f + \frac{c}{16\pi^2} \hat{K}|_0 - \sum_r \frac{T(r)}{8\pi^2} \ln \det \mathbf{Z}^{(r)}|_0 + \frac{T(G)}{8\pi^2} \ln \frac{1}{g_{phys}^2}, \quad (6)$$

$$\frac{2M}{g_{phys}^2} = (F^A \partial_A f + \frac{c}{8\pi^2} F^A \hat{K}_A - \sum_r \frac{T_a(r)}{8\pi^2} F^A \partial_A \ln \det \mathbf{Z}^{(r)}|_0) \times (1 - \frac{T(G_a)}{8\pi^2} g_{phys}^{(a)2})^{-1}. \quad (7)$$

It should be stressed that the three formulae (4)(6)(7) for the soft mass (and analogous formulae for the $\mu, B\mu$ and A terms), the gauge coupling and gaugino mass, are all expressions valid at whatever scale the explicit expressions for the Kaehler potential K as a function of the moduli Φ is given. Thus if K is obtained from string theory (after incorporating α' and string loop corrections), one expects these expressions to be valid at some point close to the string scale. These formulae are then to be used as the boundary conditions for renormalization group (RG) evolution. To one loop order the RG evolution values of the coupling function and the gaugino mass at some scale μ , would be given in terms of the value at the (Φ independent) boundary scale Λ ($\lesssim M_{string}$ if the fundamental theory is string theory), by making the replacements

$$f \rightarrow f + (b/16\pi^2) \ln(\Lambda^2/\mu^2) \quad (8)$$

and $g_{phys}^{-2} \rightarrow f$ on the RHS of (6). Note that to this order the second equation is unchanged and in fact the factor in parenthesis in the last term on the RHS can be replaced by unity. However the above formulae can actually be interpreted as being valid to all orders in the loop expansion, provided in addition to the replacement for f (8) one replaces $g_{phys}^2 \rightarrow g_{phys}^2(\mu^2)$ and

$$\mathbf{Z}^{(r)}(\Phi, \bar{\Phi}) \rightarrow \mathbf{Z}^{(r)}(\Phi, \bar{\Phi}; g^2(\mu)). \quad (9)$$

Thus we have the following formulae for the parameters at the infra red RG scale μ , which are expected to be valid to all orders in the loop expansion:

$$\begin{aligned} \frac{1}{g_{phys}^2}(\Phi, \bar{\Phi}; \mu) &= \Re f + \frac{b}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} + \frac{c}{16\pi^2} \hat{K}|_0 - \sum_r \frac{T(r)}{8\pi^2} \text{tr} \ln \mathbf{Z}^{(r)}(g^2(\mu))|_0 \\ &\quad + \frac{T(G)}{8\pi^2} \ln \frac{1}{g_{phys}^2(\mu)}, \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{2M}{g_{phys}^2}(\Phi, \bar{\Phi}; \mu) &= (F^A \partial_A f + \frac{c}{16\pi^2} F^A \hat{K}_A - \sum_r \frac{T_a(r)}{8\pi^2} F^A \partial_A \text{tr} \ln \mathbf{Z}^{(r)}(g^2(\mu))|_0) \\ &\quad \times (1 - \frac{T(G_a)}{8\pi^2} g_{phys}^{(a)2}(\mu))^{-1}. \end{aligned} \quad (11)$$

The first of these equations is the integrated form of the NSVZ beta function [12] with the boundary condition (at $\mu = \Lambda$) fixed by the KL supergravity correction (the third term above)². In fact differentiating with respect to $t \equiv \ln \mu$ and assuming the gauge neutrality of the moduli and gravitational interactions so that Φ, \hat{K} are independent of t ³ we get the NSVZ equation for the

²Note that the original derivation in [13] for the last term in (10), used the NSVZ beta function. However as shown in [14] (see also [15]), this term can be derived by arguments very similar to those used in [13] to get the third and fourth terms.

³This assumption is violated in GMSB like theories where the SUSY breaking sector is coupled via the messenger sector to the standard model gauge group.

exact beta function:

$$\frac{\beta}{g} = \frac{g^2}{16\pi^2} \frac{b + 2 \sum_r T(r) \text{tr} \gamma_r}{(1 - g^2(\mu) T(G)/8\pi^2)}, \quad (12)$$

where we've defined

$$\gamma_r = \frac{1}{2} \frac{d}{dt} \ln \mathbf{Z}^{(r)}(\Phi, \bar{\Phi}; g^2(\mu)). \quad (13)$$

(11) does not seem to have been written down explicitly before - though it is of course a straightforward consequence of the KL formula. Note its resemblance to the NSVZ beta function. Also differentiating (11) with respect to t gives the beta function for the gaugino mass

$$\beta_M \equiv \frac{dM}{dt} = \frac{2M\beta - g^3 \sum_r T(r) \text{tr} \gamma^{(1)} / 8\pi^2}{g(1 - g^2 T(G)/8\pi^2)}, \quad (14)$$

where

$$\gamma_r^{(1)} = F^A \partial_A \gamma_r \quad (15)$$

This equation has been derived earlier in [16][17], but under additional assumptions. Note that these two equations are compatible with the (expected) relation

$$\frac{dM}{dt} = -F^A \partial_A \frac{\beta}{g}, \quad (16)$$

which follows from the RG invariance of the moduli i.e.

$$[\frac{d}{dt}, F^A \partial_A] = 0, \quad (17)$$

and the formula $F^A \partial_A (1/g^2) = 2M/g^2$ (see for example [8]). This may be rewritten as,

$$F^A \partial_A g = -Mg. \quad (18)$$

What these supergravity considerations tell us, is to take the values (at the high scale Λ) for the scalar masses given by (4), along with the analogous formulae for the $\mu, B\mu$ and A terms as well as the formulae (6)(7) for the gauge couplings and gaugino mass, as boundary values for integrating the RG equations of the MSSM, to find the values at the scale μ . This follows the standard practice used for mSUGRA for instance. This is consistent with the formula (4) once we modify the Kaehler metric for the matter fields appropriately.

Thus consider the one loop correction to the matter metric. Keeping the functional form in terms of the moduli fields in the standard one loop counter term calculation in the supersymmetric theory (see for example [18]), we have (putting $g^2 = 1/\Re f$)

$$\Delta Z_{a\bar{b}} = \frac{1}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} (Y_{acd} \bar{Y}_{\bar{b}\bar{c}\bar{x}} Z^{c\bar{c}} Z^{d\bar{d}} - 2(\Re f)^{-1} T(r) Z_{a\bar{b}}). \quad (19)$$

When this correction is made to the metric in formula (4), one gets additional one loop terms when the derivatives with respect to the moduli act on moduli dependent functions in (19). In particular the derivatives acting on the gauge coupling function will give contributions proportional to the gaugino mass squared:

$$-F^A F^{\bar{B}} \Delta R_{A\bar{B}a\bar{b}} \sim -F^A F^B \partial_A \partial_B \Delta Z_{a\bar{b}} \sim \frac{1}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} T(r) g^6 |F^A \partial_A f|^2 + \dots = \frac{1}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} T(r) g^2 |M|^2, \quad (20)$$

where M is the gaugino mass (to lowest order). This gives the term $\beta^{(m^2)} \sim g^2 |M|^2 T(r)/16\pi^2$ in the beta function for the (squared) scalar mass which drives the gaugino mediated contribution to it.

2.2 Gravity (moduli) mediation

The generic and most natural situation that arises in supersymmetry breaking (given that we need to start from SUGRA), is what is often called gravity mediated SUSY breaking. However since the actual transmission of supersymmetry breaking is through the gauge neutral fields that we have called moduli, it is more appropriate to call this moduli mediated SUSY breaking (MMSSB). In general this will lead to flavor changing neutral currents (FCNC) at a level which is disallowed by experiment, so some additional assumptions are needed. In mSUGRA (see for example [19] for a recent review) one makes the assumption of universal scalar masses and a universal A-term. This follows from a certain factorization assumption for the metric on MSSM field space and the assumption of independence of the Yukawa couplings from the SUSY breaking moduli. If the cut-off (beyond which the SUGRA needs to be replaced by string theory) is well below the Planck scale, then these assumptions are preserved by quantum corrections⁴.

2.2.1 Sequestered moduli mediation: AMSB

Here we will focus on the so-called sequestered models which give a different approach to solving the FCNC problem within MMSB. This has been called anomaly mediated supersymmetry breaking (AMSB) [21, 22]. The only anomaly that one has in a SUGRA consistently coupled to an anomaly free gauge theory (like the standard model), is the Weyl anomaly discussed in [13]. For the gaugino mass this gives an extra term (namely the second term on the RHS in (7) or (11)).⁵

As for the scalar masses and the A and $B\mu$ terms, expressions for them are derived in the AMSB literature by assuming that quantum corrections that are proportional to $\ln \Lambda/\mu$ should be replaced by $\ln \Lambda C/\mu$ where C is the Weyl compensator whose F-terms are then given a non-zero value. As shown in [15] if indeed C is the Weyl compensator this procedure will actually violate the Weyl invariant formalism, and in fact the compensator will become a propagating field. On the other hand one may regard this insertion as a spurion, in which case what we have is an explicit breaking of SUSY. In any case one gets an elegant set of formulae which have the added benefit of being RG invariant. The relations in question are (see for example [23] where these are shown to be RG invariant);

$$M = M_0 \frac{\beta}{g}, \quad (21)$$

$$(m^2)_b^a = -|M_0|^2 \frac{d\gamma_b^a}{dt} = -|M_0|^2 \beta \frac{d\gamma_b^a}{dg}, \quad (22)$$

$$A_{abc} = -M_0 \frac{dY_{abc}}{dt}, \quad (23)$$

$$B_{ab} = -M_0 \frac{d\mu_{ab}}{dt}. \quad (24)$$

Here M_0 is a constant mass parameter. For completeness, we have included the expressions for the A and B terms, but let us just focus on the first two equations. Firstly we see that identifying $M_0 = F^\phi$ we have the AMSB formula for the scalar mass. In the original version of AMSB this is identified with $m_{3/2}$, the gravitino mass.

⁴For a recent discussion of these issues see [20].

⁵The question of whether there is yet another “anomaly” term proportional to the gravitino mass as is claimed in much of the phenomenological literature has been addressed and answered in the negative in [15]. The essence of the argument there was that any such claim implies that quantum effects break supersymmetry. Subsequent to the publication of that paper several authors have claimed (in effect) that even if it is absent in the Wilsonian action it will be present in the 1PI action. These arguments will be addressed in a separate publication.

Let us now show that these formulae have nothing to do with the Weyl anomaly. Writing

$$F^A \partial_A = F^A \partial_A|_g + F^A \partial_A g \partial_g|_\Phi, \quad (25)$$

we see that

$$F^A \partial_A \beta = \frac{2 \sum_r T(r) F^A \partial_A|_g \text{tr} \gamma}{1 - g^2 T(G)/8\pi^2} + F^A \partial_A g \partial_g|_\Phi \beta,$$

where we've used (12) in the first term. Now if this term can be ignored we see that (using (18)) $F^A \partial_A \beta = -Mg d\beta/dg$,

$$\frac{d}{dt} \left(\frac{Mg}{\beta} \right) = 0 \Rightarrow \left(\frac{Mg}{\beta} \right) = M_0, \quad (26)$$

where M_0 is a RG invariant constant. This is precisely the AMSB formula (now derived without inserting Weyl compensator fields) but with the above assumption. For the scalar mass we get (for simplicity we work with one family of matter fields Q), after tuning the CC to zero,

$$\begin{aligned} m^2 &= m_{3/2}^2 - F^{\bar{A}} \partial_A|_g F^B \partial_B|_g \ln Z(\Phi, \bar{\Phi}; g) \\ &\quad - (\bar{M} g \partial_g|_\Phi F^B \partial_B|_g + F^{\bar{A}} \partial_A|_g M g \partial_g|_\Phi) \ln Z(\Phi, \bar{\Phi}; g) \\ &\quad - 2 \bar{M} g \partial_g|_\Phi \frac{Mg}{\beta} \gamma, \end{aligned} \quad (27)$$

where we've put $\gamma = \frac{1}{2} d \ln Z / dt$ and used $\partial_g|_\Phi = \beta^{-1} d/dt$. Using (26) we then have for the last line,

$$\Delta m^2 = -2 \frac{|M|^2 g^2}{\beta} \frac{d\gamma}{dg}|_\Phi = -2\beta |M_0|^2 \frac{d\gamma}{dg}|_\Phi. \quad (28)$$

This is precisely the AMSB formula for the scalar masses! However clearly this is not a necessary consequence of sequestering (otherwise sequestering would be disastrous). The first line for example may be set equal to zero in sequestered models, but this can be done only at some fixed UV scale (say Λ) where the last factor in the second term has its classical value. Below this scale this will change and then this line will give non-zero contributions. The second line will also in general be non-zero even in sequestered models. Finally we had to use (26) to arrive at (28).

The key assumption that would lead to both (26) and (28) is the factorization of the kinetic function for matter fields i.e. $Z(\Phi, \bar{\Phi}; g(\mu)) = Z_0(\Phi, \bar{\Phi}) Z_1(g(\mu))$ and (for the scalar mass case) that the classical contribution is negligible (sequestering). This factorization assumption will lead to the second line of (27) becoming zero, and the second term on the first line becoming independent of g . So if the classical contribution (i.e. the value at the UV scale) is zero (i.e. sequestering) then the first line is also zero, thus giving the so-called AMSB expression (28) as the sole contribution.

Similar arguments can be made for the A and $B\mu$ terms.

2.2.2 Sequestered moduli mediation: inoAMSB

As we argued above sequestering alone does not lead to the AMSB formulae. Additional assumptions are needed and in fact they lead to disastrous consequences as is well known. What then is the most natural moduli mediated scenario, once the low energy theory inherited from the ultra violet theory (string theory) is of the sequestered form? Our claim is that this is the one discussed in [6, 7]. It may be viewed as the correct form of what is traditionally known as AMSB. Thus in this case the gaugino mass is essentially given by the anomaly terms in the KL formula, since the classical contribution is highly suppressed (sequestered). The classical scalar masses (i.e. the mass at the UV scale Λ), are all negligible due to sequestering. The masses at the low energy (gravitino)

scale are then generated by RG running and acquire non-zero values (with positive definite values for squarks and sleptons) while giving the necessary electro-weak breaking contributions to the Higgs mass matrix by RG running. The dominant contribution to this running is what has been called the gaugino mediation mechanism [24, 25].

The string theory basis for inoAMSB, was discussed in [6] and its phenomenological consequences in [7], so we will not pursue it further here. The main point that we wish to emphasize is that regardless of its origins in string theory, given a sequestered scenario, the resulting model of SUSY breaking with no additional assumptions is inoAMSB.

2.3 GMSB

In GMSB typically there is a SUSY breaking sector and a messenger sector (which in direct mediation would be part of the former), in addition to the MSSM. If the theory is embedded in string theory, then in addition there could be a low energy string theoretic moduli sector as well. However in order to construct a viable GMSB model (avoiding additional fine tuning), the latter sector should be integrated out in a (Minkowski) supersymmetric fashion ([4]). Furthermore the lowest modulus mass should be greater than the scale of the SUSY breaking sector. Although the corresponding string vacua appear to be rather sparse, in principle this is achievable. In such a situation we can focus on a SUSY breaking sector characterized by some UV scale $\Lambda \ll M_P$, which would be the mass of the lightest modulus that has been integrated out. The potential in the SUSY breaking sector is then expected to have a (meta-stable) minimum with $X = X_0 \ll \Lambda$ and $|F^X| = \sqrt{3}m_{3/2}M_P \neq 0$, where the first relation comes from tuning the CC to zero after SUSY breaking. The SUSY breaking field X couples to the messengers through a superpotential term of the form $\Delta W = \lambda X f \tilde{f}$. In principle there may be additional SUSY breaking fields

In such a situation the KL formula (10) for the gauge coupling function undergoes a modification below the messenger scale due to the messenger threshold ([26]:

$$\begin{aligned} \frac{1}{g_{phys}^2}(\Phi, \bar{\Phi}; \mu) &= \Re f + \left(\frac{b_{>}}{16\pi^2}\right) \ln\left(\frac{\Lambda^2}{X\bar{X}}\right) + \left(\frac{b_{<}}{16\pi^2}\right) \ln\left(\frac{X\bar{X}}{\mu^2}\right) + \frac{c}{16\pi^2} \hat{K}|_0 - \sum_r \frac{T(r)}{8\pi^2} \text{tr} \ln \mathbf{Z}^{(r)}(g^2(\mu))|_0 \\ &\quad + \frac{T(G)}{8\pi^2} \ln \frac{1}{g_{phys}^2(\mu)}, \end{aligned} \quad (29)$$

$$\begin{aligned} \frac{2M}{g_{phys}^2}(\Phi, \bar{\Phi}; \mu) &= (F^A \partial_A f + \frac{T_{mess}}{16\pi^2} \frac{F^X}{X} + \frac{c}{16\pi^2} F^A \hat{K}_A - \sum_r \frac{T(r)}{8\pi^2} F^A \partial_A \text{tr} \ln \mathbf{Z}^{(r)}(g^2(\mu))|_0) \\ &\quad \times (1 - \frac{T(G)}{8\pi^2} g_{phys}^2(\mu))^{-1}. \end{aligned} \quad (30)$$

Now the GMSB effect becomes relevant only in situations that the gravitino mass has been tuned to be several orders of magnitude below the soft/weak scale. Now all terms on the RHS in the expression for the gaugino mass (11) except the second are $O(m_{3/2})$, or (in the case of the $\ln Z$ term on the first line) of higher order in perturbation theory, while the second term is $O(m_{3/2}M_P/X_0) \gg m_{3/2}$ (since $X_0 \ll \Lambda$). Thus we have the usual GMSB expression

$$\frac{2M}{g_{phys}^2}(\Phi, \bar{\Phi}; \mu) \simeq + \frac{T_{mess}}{16\pi^2} \frac{F^X}{X}. \quad (31)$$

Note that (30) remains valid at all loop order even in the presence of the messenger threshold. In fact in addition to the second term on the RHS (which is necessarily a one-loop effect), there is

an additional messenger threshold effect in the third term coming from the term in the sum when $\Phi^A = X$. In this case (if $m_{3/2}$ is tuned to be sufficiently small) the messenger threshold gives the dominant contribution to this sum;

$$\sum_r \frac{T(r)}{8\pi^2} F^A \partial_A \text{tr} \ln \mathbf{Z}^{(r)}(g^2(\mu))|_0 \simeq \frac{T_{\text{mess}}}{8\pi^2} F^X \partial_X \text{tr} \ln \mathbf{Z}^{(r)}(g^2(\mu)) = 2 \frac{T_{\text{mess}}}{8\pi^2} \frac{F^X}{X} (\gamma(\mu) - \gamma(X)) \frac{\beta_{>} - \beta_{<}}{\beta_{<}}|_X. \quad (32)$$

Note that this term is of higher loop order than (31). For $\gamma = c\alpha/4\pi$ it is a factor $c\alpha \ln(X/\mu)/16\pi^3$ smaller than the leading contribution (31). It may be of relevance in situations where the leading contribution vanishes [3] as in some direct mediation models.

Similar arguments can be applied to the soft masses. Thus we may write, again using (25) in (5)

$$\begin{aligned} m^2 &= m_{3/2}^2 - F^{\bar{A}} \partial_A|_g F^B \partial_B|_g \ln Z(\Phi, \bar{\Phi}; g) \\ &\quad - (F^{\bar{A}} \frac{\partial g}{\partial \Phi^{\bar{A}}} \partial_g|_{\Phi} F^B \partial_B|_g + F^{\bar{A}} \partial_A|_g F^B \frac{\partial g}{\partial \Phi^{\bar{B}}} \partial_g|_{\Phi}) \ln Z(\Phi, \bar{\Phi}; g) \\ &\quad - F^{\bar{A}} \frac{\partial g}{\partial \Phi^{\bar{A}}} \partial_g|_{\Phi} F^B \frac{\partial g}{\partial \Phi^{\bar{B}}} \partial_g|_{\Phi} \ln Z(\Phi, \bar{\Phi}; g). \end{aligned} \quad (33)$$

Note that now because of the existence of the messenger threshold, we cannot any longer just use (26). When the sum over A gets to $\Phi^A = X$ (i.e. is one of the fields which break SUSY and couples to the messengers) one gets additional contributions from the last term in (33). For $\mu \rightarrow X_0$ this contribution is given by the standard GMSB expression,

$$m^2 \sim 2 \left| \frac{F^X}{X} \right|^2 (\beta_{>} - \beta_{<}) \frac{\partial \gamma}{\partial g} \simeq 2 \left| \frac{F^X}{X} \right|^2 T_{\text{mess}} c \left(\frac{\alpha}{4\pi} \right)^2. \quad (34)$$

In the last relation we've used the one loop beta function and anomalous dimension $\gamma = c\alpha/4\pi$. This term is of course always positive and is the standard GMSB formula. The point that we wish to emphasize here is that unlike in the case of AMSB, here the suppression of gravity mediated effects (effectively $F^X/X_0 \sim m_{3/2} M_P/X_0 \gg m_{3/2}$) means that all other terms in (33) are suppressed compared to (34). Thus with this one assumption (namely $m_{3/2} \ll M_W$) we get the GMSB formulae from the general framework.

3 Conclusions

We have given a unified treatment of currently popular phenomenological models of supersymmetry breaking, within the framework of the general theory developed in the nineties. The main point of our analysis is that these general arguments are valid even at the quantum level, provided we take into account the necessary corrections to the supergravity potentials (K, W) and the gauge coupling function f . This enables us to understand where the so-called AMSB formulae come from. These arguments also show that the natural consequence of sequestering, is the mediation mechanism that has been called inoAMSB. Finally we showed how the GMSB formulae emerge from this framework, and pointed out an additional term that has not been generally discussed in the literature.

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